Solutions of Additional Practice problems

Topic I

Use the following to answer questions 1 and 2:

You have been given this probability distribution for the holding period return for XYZ stock:

State of the Economy	Probability	HPR (%)
Boom	0.30	18
Normal growth	0.50	12
Recession	0.20	-5

1. What is the expected holding period return for XYZ stock?

A) 11.67%

B) 8.33%

C) 10.4%

D) 12.4%

E) 7.88%

Solution.

$E(r_{XYZ}) = .3 \times 18 + .5 \times 12 + .2 \times (-5) = 10.4\%$

2. What is the expected standard deviation for XYZ stock?

A) 2.07%

B) 9.96%

C) 7.04%

D) 1.44%

E) 8.13%

Solution.

$\sigma(r_{XYZ}) = \sqrt{.3 \times (18 - 10.4)^2 + .5 \times (12 - 10.4)^2 + .2 \times (-5 - 10.4)^2} = 8.13\%$

3. The standard deviation of return on investment A is 40% while the standard deviation of return on investment B is 20%. What is the correlation coefficient

between returns on A and B if the covariance of returns on A and B is 0.050?

Solution. $\rho = \frac{cov(r_A, r_B)}{\sigma_A \sigma_B} = \frac{0.05}{0.2 \times 0.4} = 0.625.$

4. Consider a portfolio with 70% in stock A and 30% in stock B.

Stock A: $E(r_A) = 0.15, \quad \sigma_A = 0.20$

Stock B: $E(r_B) = 0.10, \quad \sigma_B = 0.5$

$\rho_{AB} = 0.05$

Find the expected return and the standard deviation of the portfolio returns

Solution.

$$E(r_P) = wE(r_A) + (1-w)E(r_B) = .7 \times .15 + .3 \times .10 = .135$$

$$\sigma_P = \sqrt{w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B\rho_{AB}} =$$

$$\sqrt{.7^2 \times .2^2 + .3^2 \times .5^2 + 2 \times .7 \times .3 \times .2 \times .5 \times .05} = .21$$

5. In a return-standard deviation space, which of the following statements is (are) true for risk-averse investors? (The vertical and horizontal lines are referred to as the expected return-axis and the standard deviation-axis, respectively.)

I) An investor's own indifference curves might intersect.

II) Indifference curves have negative slopes.

III) In a set of indifference curves, the highest offers the greatest utility.

IV) Indifference curves of two investors might intersect.

A) I and II only

B) II and III only

C) I and IV only

D) III and IV only

E) none of the above

Solution.

D), since an investor's own indifference curves can not intersect and they are upward sloping

6. Suppose you currently hold a portfolio that yields 15% return and has a standard deviation of 20%. What should the minimum rate of return on a portfolio with standard deviation of 25% be to make you indifferent between the two portfolios? Assume your coefficient of risk aversion is 3.

Solution.

Investor is indifferent between the two portfolios if they provide the same utility.

Utility from the first portfolio:

$U_1 = .15 - \frac{1}{2} \times 3 \times (.2)^2 = .09$

Utility from the second portfolio:

$U_2 = E(r_2) - \frac{1}{2} \times 3 \times (.25)^2 = E(r_2) - .09375$

Therefore, an investor is indifferent between the two portfolios if $E(r_2) - .09375 = .09$, or $E(r_2) = .18375$

7. A portfolio is comprised of two stocks, A and B. Stock A has a standard deviation of return of 20% while stock B has a standard deviation of return of 5%. Stock A comprises 30% of the portfolio while stock B comprises 70% of the portfolio. If the variance of the return on the portfolio is 0.0230, what is the correlation coefficient between returns on A and B?

Solution.

The variance of the whole portfolio is

$$\sigma_P^2 = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B\rho_{AB}$$

Therefore,

$$\rho_{AB} = \frac{\sigma_P^2 - w^2\sigma_A^2 - (1-w)^2\sigma_B^2}{2w(1-w)\sigma_A\sigma_B} = \frac{.023 - .7^2 \times .2^2 - .3^2 \times .05^2}{2 \times .7 \times .3 \times .2 \times .05} = \frac{.003175}{.0042} = 0.76$$

Topic II

1. An investor invests 30% of his wealth in a risky asset with an expected rate of return of 0.15 and a variance of 0.04 and 70% in a T-bill that pays 6%. His portfolio's expected return and standard deviation are

A) 0.114 and 0.12

B) 0.087 and 0.06

C) 0.295 and 0.12

D) 0.087 and 0.12

E) none of the above,

respectively

Solution.

$$E(r_C) = yE(r_P) + (1 - y)r_f = .3 \times .15 + .7 \times .06 = .087$$

$$\sigma_C = y\sigma_P = .3 \times .2 = 0.06$$

2. You invest \$1000 in a complete portfolio which is comprised of a risky asset with an expected rate of return of 12% and a standard deviation of 20% and a T-bill with a rate of return of 5%. If you want your complete portfolio to have a standard deviation of 10%, how much should you invest in T-bills? What would the expected return on that portfolio be?

Solution.

From $\sigma_C = y\sigma_P = y \times 0.2 = .1$ we find $y = 0.5$ and $1 - y = .5$

$$E(r_C) = yE(r_P) + (1 - y)r_f = .5 \times .12 + .5 \times .05 = .085$$

3. You have \$1,000,000 to invest. The risk-free rate as well as your borrowing rate is 2%. The return on the risky portfolio is 12%. If you wish to earn a 22% return, how much should you borrow?

Solution.

$E(r_C) = yE(r_P) + (1 - y)r_f = y \times .12 + (1 - y) \times .02 = .02 + y \times .1 = .22$ implies $y = 2$.

Therefore, investor has to borrow \$1,000,000.

5. You are considering investing \$1,000 in a complete portfolio. The complete portfolio is comprised of T-bills that pay 3% and a risky portfolio P, constructed from 2 risky securities X and Y. The weights of X and Y in P are 80% and 20%, respectively. X has expected rate of return of 15% and Y has an expected rate of return of 8%. To form a complete portfolio with an expected rate of return of 10%, how much should you invest in T -bills, security X and security Y?

Solution.

Expected return of the risky portfolio

$$E(r_P) = w_X E(r_X) + w_Y E(r_Y) = .8 \times 15 + .2 \times 8 = 13.6$$

Return of the complete portfolio

$$E(r_C) = r_f + y[E(r_P) - r_f] = 3 + y \times 10.6 = 10$$

Thus, $y = .66$, $1 - y = .34$. Fractions of the complete portfolio invested in stock X and Y are $.66 \times .8 = .528$ and $.66 \times .2 = .132$, respectively.

6. An investor can design a risky portfolio based on two stocks, K and L. Stock K has an expected rate of return of 18% and a standard deviation of return of 30%. Stock L has an expected rate of return of 14% and a standard deviation of return of 5%. The correlation coefficient between the two stocks is 0.5. The risk-free rate is 5%.

(a) what is the weight of stock K in the optimal risky portfolio?

(b) what is the expected return on the optimal risky portfolio?

(c) what is the standard deviation of return on the optimal risky portfolio?

Solution.

(a) We know

$$w_K^{\star} = \frac{[E(r_K) - r_f]\sigma_L^2 - [E(r_L) - r_f]Cov(r_L, r_K)}{[E(r_L) - r_f]\sigma_K^2 + [E(r_K) - r_f]\sigma_L^2 - [E(r_L) - r_f + E(r_K) - r_f]Cov(r_L, r_K)}$$

Substituting our data into the last equation gives

$$
\begin{aligned}
w_K^\star &= \frac{(0.18-0.05)0.05^2-(0.14-0.05)\times .3\times .05\times 0.5}{(0.14-0.05)0.3^2+(0.18-0.05)0.05^2-(0.14-0.05+0.18-0.05)\times .3\times .05\times 0.5} \\
&= = -0.052 \\
w_L^\star &= 1-(-.052)=1.052 \\
E(r_P) &= (-0.052)\times 0.18+1.052\times 0.14=0.1379 \\
\sigma_P &= \sqrt{.052^2\times 0.3^2+1.052^2\times .05^2+2\times(-0.052)\times 1.052\times .3\times .05\times 0.5}=0.0468
\end{aligned}
$$

7. When borrowing and lending at a risk-free rate are allowed, which Capital Allocation Line (CAL) should the investor choose to combine with the efficient frontier?

I) with the highest reward-to-variability ratio.

II) that will maximize his utility

III) with the steepest slope.

IV) with the lowest slope.

A) I and III

B) I and IV

C) II and IV

D) I only

E) I, II, and III

Solution.

E) The slope of CAL is the reward-to-variability ratio and it should be the highest. The highest slope also maximizes the expected utility of an investor.

Topic III

1. Which statement is not true regarding the market portfolio?

A) It includes all publicly traded financial assets.

B) It lies on the efficient frontier.

C) All securities in the market portfolio are held in proportion to their market values.

D) It is the tangency point between the capital market line and the indifference curve.

E) all of the above are true.

Solution.

D) The tangency point between the capital market line and the indifference curve is a complete portfolio of an investor which includes a risk–free asset.

2. Your personal opinion is that security X has an expected rate of return of 0.11. It has a beta of 1.5. The risk-free rate is 0.05 and the market expected rate of return is 0.09. According to the Capital Asset Pricing Model, this security is

A) underpriced.

B) overpriced.

C) fairly priced.

D) cannot be determined from data provided.

E) none of the above.

Solution. C) $\alpha = E(r_i) - [r_f + \beta_i[E(r_M) - r_f]] = 0.11 - [0.05 + 1.5(0.09 - 0.05)] = 0$

3. According to CAPM, the fair return of the stock is 15%.

(a) What is the stock's beta if the risk–free rate is 3% and the market return is 12%?

(b) What is the standard deviation of the return on the market portfolio if $cov(r_i, r_M) = 0.06$

Solution.

(a) From $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$ we find

$$\beta_i = \frac{E(r_i) - r_f}{E(r_M) - r_f}\frac{15 - 3}{12 - 3} = 1.33$$

(b) From $\beta_i = \frac{cov(r_i, r_M)}{\sigma_M^2}$ we find

$$\sigma_M^2 = \frac{cov(r_i, r_M)}{\beta_i} = \frac{0.06}{1.33} = 0.045$$

4. The fair expected return of the stock is 15%. Find the market risk premium if the risk–free rate is 5% and the stock's beta is 0.9. What is the market expected return?

Solution.

From $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$ we find

$$E(r_M) - r_f = \frac{E(r_i) - r_f}{\beta_i} = \frac{15 - 5}{0.9} = 11.1\%,$$

$E(r_M) = 11.1 + 5 = 16.1\%$

Topic IV

1. Suppose you held a well-diversified portfolio with a very large number of securities, and that the single index model holds. If the σ of your portfolio was 0.20 and σ_M was 0.16, the β of the portfolio would be approximately

A) 0.64

B) 0.80

C) 1.25

D) 1.56

E) none of the above

Solution. C): Because $\sigma^2 \approx \beta^2\sigma_M^2$, we find $\beta = \sigma/\sigma_M = .2/.16 = 1.25$

2. (This problem can be skipped as we did not covered the linear regression analysis in the class.) The index model for stock A has been estimated with the following result:

$$r_A = 0.01 + 0.9R_M + e_A$$

If $\sigma_M = 0.25$ and $R_A^2 = 0.25$, the standard deviation of return of stock A is

A) 0.2025

B) 0.2500

C) 0.4500

D) 0.8100

E) none of the above

Solution. C): Because $R^2 = \beta^2\sigma_M^2/\sigma^2$, we find $\sigma = \beta\sigma_M/R = 0.9 \times .25/.5 = .45$

3. Consider the single factor APT. Portfolio A has a beta of 0.2 and an expected return of 13%. Portfolio B has a beta of 0.4 and an expected return of 15%. The risk-free rat of return is 10%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio and a long position in portfolio

A) A, A

B) A, B

C) B, A

D) B, B

E) none of the above

Solution. C): Portfolio A: $r_A = 13\% + 0.2F$; Portfolio B: $r_B = 15\% + 0.4F$. Make portfolio C from Portfolio B and a risk free asset with equal weights. Then $\beta_C = 0.2$ and $r_C = 0.5r_B + 0.5r_f = 7.5\% + 0.2F + 5\% = 12.5\% + 0.2F$. Because portfolios A and C have the same risk but A has higher expected return, we short C (and so B) and take a long position in A.

4. Consider the multifactor APT with two factors. Stock A has an expected return of 16.4%, a beta of 1.4 on factor 1 and a beta of .8 on factor 2. The risk premium on the factor 1 portfolio is 3%. The risk-free rate of return is 6%. What is the risk-premium on factor if no arbitrage opportunities exit?

Solution. D) :

$16.4\% = 1.4(3\%) + .8x + 6\% \quad \rightarrow \quad x = 7.75.$

Use the following to answer questions 5 and 6:

Consider the multifactor APT. There are two independent economic factors, F_1 and F_2. The risk-free rate of return is 6%. The following information is available about two well-diversified portfolios:

Portfolio	β on F_1	β on F_2	Expected Return (%)
A	1.0	2.0	19
B	2.0	0.0	12

5. Assuming no arbitrage opportunities exist, the risk premium on the factor F_1 portfolio should be

A) 3%

B) 4%

C) 5%

D) 6%

E) none of the above

Solution. A: 2A: $38\% = 12\% + 2.0(RP_1) + 4.0(RP_2)$; B: $12\% = 6\% + 2.0(RP_1) + 0.0(RP_2)$; Let us solve the last two equations with respect to RP_1 and RP_2.

By taking the difference between the two we find $26\% = 6\% + 4.0(RP_2)$; $\rightarrow$ $RP_2 = 5$;. Now from portfolio A expected return: $19\% = 6\% + RP_1 + 2.0(5)$; we arrive at $RP_1 = 3\%$.

6. Assuming no arbitrage opportunities exist, the risk premium on the factor F_2 portfolio should be

A) 3%

B) 4%

C) 5%

D) 6%

E) none of the above

Solution. C): See the previous solution

Topic V

1. a. If a manager is not allowed to sell short he will not include stocks with negative alphas in his portfolio, so that A and C are the only ones he will consider.

	α	σ_e^2	$\frac{\alpha}{\sigma_e^2}$	$\frac{\alpha/\sigma_e^2}{\sum \alpha/\sigma_e^2}$
A:	1.6	3364	.000476	.3352
C:	3.4	3600	$\underline{.000944}$	$\underline{.6648}$
			.001420	1.0000

The forecast for the active portfolio is:

$$\alpha = .33521.6 + .66483.4 = 2.80\%$$

$$\beta = .33521.3 + .66480.7 = 0.90$$

$$\sigma_e^2 = .335223364 + .664823600 = 1969.03$$

$$\sigma_e = 44.37\%$$

The weight in the active portfolio is:

$$w_0 = \frac{\alpha/\sigma_e^2}{E(R_M)/\sigma_M^2} = \frac{2.80/1969.03}{8/23^2} = .0940$$

and adjusting for beta

$$w^* = \frac{w_0}{1+(1-\beta)w_0} = \frac{.094}{1+(1-.90)(.094)} = .0931$$

The appraisal ratio of the active portfolio is

$$A = \alpha/\sigma_e = 2.80/44.37 = .0631$$

and hence, the square of Sharpe's measure is:

$$S^2 = (8/23)^2 + .0631^2 = .1250$$

and S = .3535, compared to the market's Sharpe measure $S_M = .3478$. When short sales were allowed (problem in the home assignment), the manager's Sharpe measure was higher, .3662. The reduction in the Sharpe measure is the cost of the short sale restriction.

The characteristics of the optimal risky portfolio are:

$$\beta_P = w_M + w_A\beta_A = 1 - .0931 + .0931 \times .9 = .99$$

$$E(R_P) = P + \beta_P E(R_M) = .0931 \times 2.8 + .99 \times 8 = 8.18\%$$

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_{e_P}^2 = (.99 \times 23)^2 + .0931^2 \times 1969.03 = 535.54$$

$$\sigma_P = 23.14\%$$

With A = 2.8, the optimal position in this portfolio is:

$$y = \frac{8.18}{.01 \times 2.8 \times 535.54} = .5455$$

The final positions in each asset are:

$$\text{Bills:} \quad 1 - .5455 = 45.45\%$$

$$\text{M:} \quad .5455(1 - .0931) = 49.47\%$$

$$\text{A:} \quad .5455 \times .0931 \times .3352 = 1.70\%$$

$$\text{C:} \quad .5455 \times .0931 \times .6648 = 3.38\%$$

b. The mean and variance of the optimized complete portfolios in the unconstrained and short-sales constrained cases, and the passive strategy are:

	$E(R_C)$	σ_C^2
Unconstrained	$.5685 \times 8.42 = 4.79$	$.56852 \times 528.93 = 170.95$
Constrained	$.5455 \times 8.18 = 4.46$	$.54552 \times 535.54 = 159.36$
Passive	$.5401 \times 8.00 = 4.32$	$.54012 \times 529.00 = 154.31$

The utility level, $E(r_C) - .005A\sigma_C^2$ is:

Unconstrained	$8 + 4.79 - .005 \times 2.8 \times 170.95 = 10.40$
Constrained	$8 + 4.46 - .005 \times 2.8 \times 159.36 = 10.23$
Passive	$8 + 4.32 - .005 \times 2.8 \times 154.31 = 10.16$

2. The Treynor-Black model assumes that

A) the objective of security analysis is to form an active portfolio of a limited number of mispriced securities.

B) the cost of less than full diversification comes from the nonsystematic risk of the mispriced stock.

C) the optimal weight of a mispriced security in the active portfolio is a function of the degree of mispricing, the market sensitivity of the security, and its degree of nonsystematic risk.

D) all of the above are true.

E) none of the above are true.

Solution. D

Topic VI

1. A coupon bond that pays interest of \$100 annually has a par value of \$1,000, matures in 5 years, and is selling today at a \$72 discount from par value. The yield to maturity on this bond is

A) 6.00 %

B) 8.33 %

C) 12.00 %

D) 60.00 %

E) none of the above

Solution. C. Check: $\frac{100}{1.12} + \frac{100}{1.12^2} + ... + \frac{100}{1.12^5} + \frac{1000}{1.12^5} = \$927.9$

2. You purchased an annual interest coupon bond one year ago that had 6 years remaining to maturity at that time. The coupon interest rate was 10% and the par value was \$1,000. At the time you purchased the bond, the yield to maturity was 8%. If you sold the bond after receiving the first interest payment and the yield to maturity continued to be 8%, your annual total rate of return on holding the bond for that year would have been

A) 7.00 %

B) 7.82 %

C) 8.00 %

D) 11.95 %

E) none of the above

Solution. C: Price of the bond one year from now is

$$P_1 = \frac{100}{1.08} + \frac{100}{1.08^2} + ... + \frac{100}{1.08^5} + \frac{1000}{1.08^5} = \$1079.85$$

Price of the bond today is

$$P_1 = \frac{100}{1.08} + \frac{100}{1.08^2} + ... + \frac{100}{1.08^5} + \frac{100}{1.08^6} + \frac{1000}{1.08^6} = \$1092.46$$

$HPR = \frac{1{,}079.85 - 1{,}092.46 + 100}{1{,}092.46} = 8\%$

Use the following to answer questions 3-6:

The following is a list of prices for zero coupon bonds with different maturities and par value of $\$1,000$.

Maturity (Years)	Price ($)
1	943.40
2	881.68
3	808.88
4	742.09

3. What is, according to the expectations theory, the expected forward rate in the third year?

A) 7.00 %

B) 7.33 %

C) 9.00 %

D) 11.19 %

E) none of the above

Solution. C: $881.68/808.88 - 1 = 9\%$

4. What is the yield to maturity on a 3-year zero coupon bond?

A) 6.37 %

B) 9.00 %

C) 7.33 %

D) 10.00 %

E) none of the above

Solution. C: $(1000/808.81)^{1/3} - 1 = 7.33\%$

5. What is the price of a 4-year maturity bond with a 12% coupon rate paid annually? (Par value $= \$1,000$)

A) $\$742.09$

B) $\$1,222.09$

C) $\$1,000.00$

D) $\$1,141.84$

E) none of the above

Solution. E. Interest rates: $y_1 = (1000/943.40) - 1 = 6.00\%$, $y_2 = (1000/881.68)^{1/2} - 1 = 6.50\%$, $y_3 = (1000/808.88)^{1/3} - 1 = 7.33\%$, $y_4 = (1000/742.09)^{1/4} - 1 = 7.742\%$

Price $P = \frac{120}{1.06} + \frac{120}{1.065^2} + \frac{120}{1.0733^3} + \frac{1120}{1.0742^4} = \$1157.22$

6. Consider two annual coupon bonds, each with two years to maturity. Bond A has a 7% coupon and a price of $\$1,000.62$. Bond B has a 10% coupon and sells for $\$1,055.12$. Find the two one-period forward rates that must hold for these bonds.

A) 6.97%, 6.95%

B) 6.95%, 6.95%

C) 6.97%, 6.97%

D) 6.08%, 7.92%

E) 7.00%, 10.00%

Solution. D. One has to solve

$$1,000.62 = d_1 \times 70 + d_2 \times 1,070$$

$$1,055.12 = d_1 \times 100 + d_2 \times 1,100$$

where $d_1 = 1/(1 + r_1)$, $d_2 = 1/(1 + r_2)^2$

We find $d_2 = .8735; d_1 = .9427$ which implies $r_1 = 6.08\%; r_2 = 7.00\%$. Hence the forward rates are $r_1 = 6.08\%$ and $f_2 = (1.07)^2/1.06 = 7.92\%$

7. Holding other factors constant, which one of the following bonds has the smallest price volatility?

A) 5-year, 0% coupon bond

B) 5-year, 12% coupon bond

C) 5 year, 14% coupon bond

D) 5-year, 10% coupon bond

E) Cannot tell from the information given.

Solution. C: Duration (and thus price volatility) is lower when the coupon rates are higher.

8. The duration of a par value bond with a coupon rate of 8% and a remaining time to maturity of 5 years is

A) 5 years.

B) 5.4 years.

C) 4.17 years.

D) 4.31 years.

E) none of the above.

Solution. D

Calculations are shown below.

Yr.	CF, $	PV of CF at 8%, $	Weight * Yr.
1	80	$80/1.08 = 74.07$	$0.0741 * 1 = 0.0741$
2	80	$80/1.08^2 = 68.59$	$0.0686 * 2 = 0.1372$
3	80	$80/1.08^3 = 63.51$	$0.0635 * 3 = 0.1905$
4	80	$80/1.08^4 = 58.80$	$0.0588 * 4 = 0.2352$
5	1080	$1080/1.08^5 = 735.03$	$0.735 * 5 = 3.6750$
Sum		$1000	4.312 years

9. Which one of the following par value 12% coupon bonds experiences a price

change of \$23 when the market yield changes by 50 basis points?

A) The bond with a duration of 6 years.

B) The bond with a duration of 5 years.

C) The bond with a duration of 2.7 years.

D) The bond with a duration of 5.15 years.

E) None of the above.

Solution. D:

$$\Delta P/P = -D \times [\Delta(1+y)/(1+y)]$$

$$-.023 = -D \times [.005/1.12]; D = 5.15.$$

10. You have an obligation to pay \$1,488 in four years and 2 months. In which bond would you invest your \$1,000 to accumulate this amount, with relative certainty, even if the yield on the bond declines to 9.5% immediately after you purchase the bond?

A) a 6-year; 10% coupon par value bond

B) a 5-year; 10% coupon par value bond

C) a 5-year; zero-coupon bond

D) a 4-year; 10% coupon par value bond

E) none of the above

Solution. B

When duration = horizon date, one is immunized, or protected, against one interest rate change. The zero has D = 5. Since the other bonds have the same coupon and yield, solve for the closest value of T that gives D = 4.2 years.

Because the duration of a corporate bond is

$$\frac{1+y}{y} - \frac{(1+y)+T(c-y)}{c[(1+y)^T-1]+y},$$

we solve an equation

$$4.2 = \frac{1.1}{.1} - \frac{(1.1+T(.1-.1)}{.1[1.1^T-1]+.1}$$

or $[ln(1.10)] = ln(1.6176); T = 5.05$ years, so choose the 5-year 10% coupon bond.

Topic VII

1. Buyers of put options anticipate the value of the underlying asset will and sellers of call options anticipate the value of the underlying asset will

A) increase; increase

B) decrease; increase

C) increase; decrease

D) decrease; decrease

E) cannot tell without further information

Solution. D: The buyer of the put option hopes the price will fall in order to exercise the option and sell the stock at a price higher than the market price. Likewise, the seller of the call option hopes the price will decrease so the option will expire worthless.

2. HighFlyer Stock currently sells for $48. A one-year call option with strike price of $55 sells for $9, and the risk free interest rate is 6%. What is the price of a one-year put with strike price of $55?

A) $9.00

B) $12.89

C) $16.00

D) $18.72

E) $15.60

Solution. B: We find from the put–call parity $P = 9 - 48 + 55/(1.06); P = 12.89$

3. Use the two-state put option value in this problem. $S_0 = \$100; X = \$120$; the two possibilities for S_T are $150 and $80. The range of P across the two states is ; the hedge ratio is

A) $0 and $40; -4/7

B) \$0 and \$50; +4/7

C) \$0 and \$40; +4/7

D) \$0 and \$50; -4/7

E) \$20 and \$40; +1/2

Solution. A: When $S_T = \$150; P = \$0$; When $S_T = \$80 : P = \$40$. Thus, $(\$0 - \$40)/(\$150 - \$80) = -4/7$.

4. A portfolio consists of 400 shares of stock and 200 calls on that stock. If the hedge ratio for the call is 0.6, what would be the dollar change in the value of the portfolio in response to a one dollar decline in the stock price?

A) +\$700

B) +\$500

C) -\$580

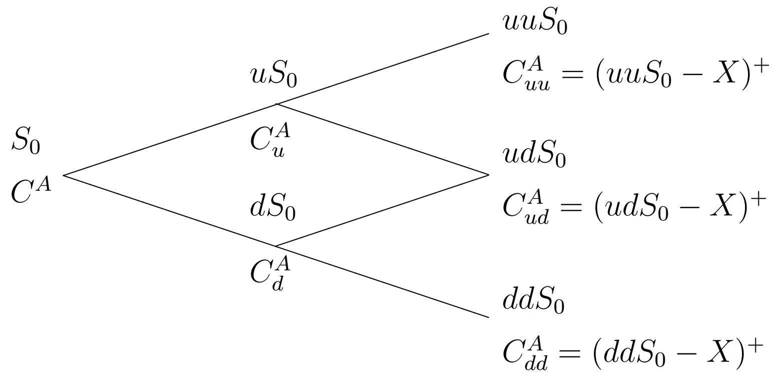
D) -\$520

E) none of the above

Solution. D: $-\$400 + [-\$200(0.6)] = -\$520$.

5. 5. Suppose that $S_0 = \$50$, $d = 0.75$, $u = 1.25$, $X = \$50$, $T = 0.5$, and the effective annual risk–free rate is $r_f = 5\%$. Find the price of the American call option today by using the two period binomial model. Assume that the stock does not pay dividends.

Solution. Consider the diagram below.



First, we find $C_{uu}^A = (1.25^2 \times 50 - 50)^+ = 28.125$, $C_{ud}^A = (1.25 \times 0.75 \times 50 - 50)^+ = 0$, $C_{dd}^A = (0.75^2 \times 50 - 50)^+ = 0$

Let us look at the node uS_0. The stock price at this node is $uS_0 = 62.5$ and the value of option exercising is $(uS_0 - X)^+ = 12.5$. If option is not exercised, then its value is

$$C_u = \frac{C_{uu}^A(1 - d + r_f) + C_{ud}^A(u - 1 - r_f)}{(u-d)(1+r_f)} = \frac{28.125(1 + 0.01227 - .75)}{(1.25 - 0.75)(1 + 0.01227)} = \$14.57$$

It follows that the option price, C_u^A, at node uS_0 is \$14.57 and the option should not be exercised.

Now let us consider the node dS_0. If put is exercised then price is $(dS_0 - 50)^+ = 0$. If option is not exercised then its value is

$$C_d = \frac{C_{ud}^A(1 - d + r_f) + C_{dd}^A(u - 1 - r_f)}{(u-d)(1+r_f)} = 0$$

It follows that the price, C_d^A, at node dS_0 is 0 no matter if the option is exercised or not.

Finally, let us consider the node S_0. If option is exercised then the payoff is 0. If not then its value is

$$C = \frac{C_u^A(1 - d + r_f) + C_d^A(u - 1 - r_f)}{(u-d)(1+r_f)} =$$

$$\frac{14.57(1. - .75 + 0.01227)}{(1.25 - 0.75)(1 + 0.01227)} = \$7.55.$$

It follows that the no–arbitrage price today, C^A, should be \$7.55 and the option should not be exercised.

Notice that the option should not be exercised in any of the node before expiration. It follows that the price of American call option is equal to the price of European option given that the stock does not pay dividends.